

# **$CP$ Violation via Top Quark Anomalous Interaction at the Fermilab Tevatron**

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**ABSTRACT:** Expecting the forthcoming experiment at the upgraded Fermilab Tevatron, we calculated  $CP$ -violating polarization asymmetry of  $t\bar{t}$ ,  $\mathcal{A}_{CP} \equiv [\sigma(p\bar{p} \rightarrow t(-)\bar{t}(-)X) - \sigma(p\bar{p} \rightarrow t(+)\bar{t}(+)X)] / \sigma(p\bar{p} \rightarrow t\bar{t}X)$ , due to possible anomalous chromomagnetic ( $\kappa$ ) and chromoelectric ( $\tilde{\kappa}$ ) couplings of the gluon to the top quark. Since  $\kappa$  and  $\tilde{\kappa}$  are sensitive to a contribution from new physics beyond standard model, this observable is useful to search for a signal of new physics. It was seen that the magnitude of  $\mathcal{A}_{CP}$  depends only on  $\text{Im}(\tilde{\kappa})$  and  $\text{Im}(\kappa^*\tilde{\kappa})$ . Furthermore, we found that when  $|\text{Im}(\kappa^*\tilde{\kappa})| > 0.5$ , one can possibly detect the  $CP$ -violation effect as a signal of new physics even if the magnitude of  $\text{Im}(\tilde{\kappa})$  is zero.

**KEYWORDS:** Top quark,  $CP$  Violation, Anomalous Interaction.

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## 1. INTRODUCTION

The Standard model (SM) which is composed of electroweak theory and quantum chromodynamics is extremely successful in particle physics phenomenology. The predictions by the SM have been in agreement with all experimental data up to the scale of  $O(M_{W/Z})$ . Recently Brookhaven E821 group reported that observed anomalous magnetic moment of positive muon was in disagreement with the SM prediction at 2.6 standard deviation [1], which seems to suggest the existence of new physics beyond SM. However, this experiment was already closed, though the statistical error might become smaller in undergoing data analysis<sup>1</sup>. Therefore, it is very important and challenging to search for a signal of new physics at forthcoming high energy collider experiments.

On the other hand, the physics of the top quark which is discovered as a very heavy particle,  $m_t \simeq 180$  GeV, at the Fermilab Tevatron with center-of-mass energy  $\sqrt{s} = 1.8$  TeV in 1994 [2], is very interesting. Since the mass of the top quark is much larger than the masses of other quarks and leptons (and even those of the electroweak gauge bosons), studies on the role of this particle in Nature is expected to lead us to the physics beyond the SM. For example, as it is well-known, the *CP*-violation in the top quark pair production is estimated to be extremely small in the

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<sup>1</sup>New measurements are now underway with negative muon which will provide a sensitive test of *CPT*-violation.

SM [3], based on the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [4, 5]. Thus, it is very interesting to search for other possible origin of  $CP$ -violation in the top quark sector originated from new physics. Furthermore, it is remarkable that due to its huge mass, the top quark decays before it hadronizes [6]. Then, we can easily get information about physics of the produced top quark from the decay distribution of secondary leptons and hadrons [7, 8]. These properties are very advantageous for searching the  $CP$ -violation in top quark sector as a signal of new physics.

Based on the above consideration, in this paper we investigate the  $CP$ -violation effect originated from new physics in the top quark pair production process at the upgraded Fermilab Tevatron. Here, we assume the existence of chromomagnetic and chromoelectric type couplings which could be sizable if new physics is in existence as a non-standard effect.

This paper is organized as follows. In Chapter 2, we introduce the  $CP$ -violating observable for the top quark pair production and the effective Lagrangian which include anomalous chromomagnetic and chromoelectric couplings of gluon to top quark. In Chapter 3, we calculate the effect of the  $CP$ -violation for  $p\bar{p} \rightarrow t\bar{t}X$  processes and discuss the results. Chapter 4 is devoted to the summary of this work and a future outlook.

## 2. $CP$ -VIOLATION IN TOP QUARK PAIR PRODUCTION

In this section, to study the contribution of the top quark anomalous chromomagnetic and chromoelectric dipole type couplings to the polarization asymmetry for the process;

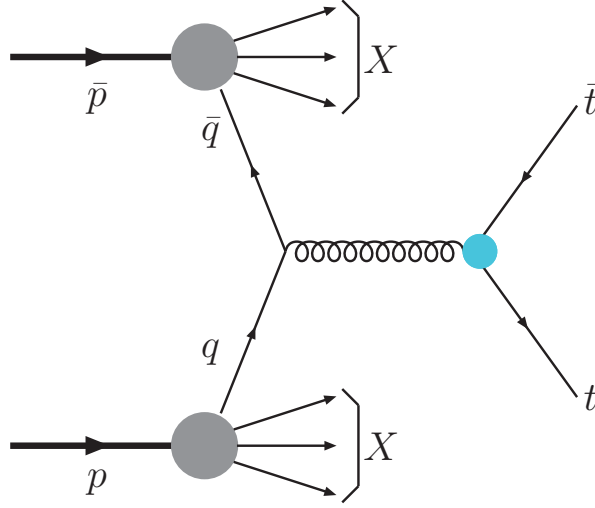
$$p\bar{p} \rightarrow t\bar{t}X, \quad (2.1)$$

which will be observed at the upgraded Fermilab Tevatron with  $\sqrt{s} = 2.0$  TeV, we introduce an observable for  $CP$ -violation and an effective Lagrangian which includes the top quark anomalous chromomagnetic and chromoelectric dipole type couplings.

Since the quark-anti-quark annihilation ( $q\bar{q} \rightarrow t\bar{t}$ ) is the major source of the top quark pair production at the Tevatron as shown in Fig. 1, we neglect subprocesses of the gluon fusion ( $gg \rightarrow t\bar{t}$ ) in this work [9].

### 2.1 $CP$ -violating Observable

Since top quark pairs are mainly produced through the gluon interaction at Tevatron (Fig. 1), the helicities of  $t\bar{t}$  would be  $(+-)$  or  $(-+)$  due to the helicity conservation which is realized if the top quark mass is much smaller than  $\sqrt{s}$ . However, since top quark mass is about 180 GeV, we can also expect to have  $(++)$  and  $(--)$  combinations as a consequence of the breaking of the helicity conservation. We can use these combinations to study  $CP$  properties of the  $t\bar{t}$  state;  $|h_t h_{\bar{t}}\rangle$ .



**Figure 1:** Feynman diagram for top quark pair production at Tevatron

$|-+\rangle$  and  $|+-\rangle$  are  $CP$  self-conjugate while  $|--\rangle$  and  $|++\rangle$  transform each other under  $CP$  operation as

$$\hat{C}\hat{P}|\mp\mp\rangle = \hat{C}|\pm\pm\rangle = |\pm\pm\rangle. \quad (2.2)$$

Therefore, the difference between the events  $N(--)$  and  $N(++)$  could be a useful measurement of  $CP$  violation [10];

$$\begin{aligned} \mathcal{A}_{cp} &= \frac{N(--)-N(++)}{N(all)} \\ &= \frac{\sigma_{--}-\sigma_{++}}{\sigma_{total}} \equiv \frac{\Delta\sigma}{\sigma_{total}}, \\ N(all) &\equiv N(++) + N(+-) + N(-+) + N(--), \end{aligned} \quad (2.3)$$

where  $\sigma_{++}$  and  $\sigma_{--}$  is the cross section of the  $t\bar{t}$  production with the helicities  $(++)$  and  $(--)$ .

Though the produced top quark can not be directly detected, we can easily reconstruct the top quark signal through the produced top quark decay distribution of the secondary leptons [7, 8]. Notice that there are some remarkable properties about the top quark decays [10];

- (1) Since top quark is very heavy ( $m_t \sim 180$  GeV) and its life is much shorter than  $10^{-23}$  s being smaller than the hadronization time, the top would decay without the hadronic effect [6].

- (2) Since  $m_t > m_W$ , the dominant decay process of the top quark should be  $t \rightarrow W^+ b$ . Because of large top mass, the  $W$  will be predominantly longitudinal while the  $b$  is always left-handed in the SM, if  $m_b/\sqrt{s} \ll 1$ <sup>2</sup>.
- (3) Because of (2), a  $t(-)$  will decay to an energetic  $b(-)$ <sup>3</sup>, which must go forward to carry the quark spin, and to a less energetic  $W^+$ ; for  $t(+)$ , the relative energies of  $b$  and  $W$  are roughly reversed.
- (4) Therefore, we can effectively get information about the polarization of the top quark by observing the energy distribution of the  $W$  bosons or their decay leptons.

In addition, as described before the  $CP$ -violation in top quark pair production is estimated to be extremely small in the CKM mechanism [3]. Thus, we have a good opportunity for investigating the non-standard origin of  $CP$ -violation in top quark sector.

In the following sections, we calculate the  $CP$ -violating observable for  $p\bar{p} \rightarrow t\bar{t}X$  at Tevatron by using effective Lagrangian being introduced in the following subsection.

## 2.2 Effective Lagrangian

In order to estimate the effect of  $CP$ -violation in the top quark pair production at Tevatron, we take the following effective Lagrangian for top-quark-gluon interaction:

$$L_{t\bar{t}g} = g_s T^a \bar{v}_t \left[ -\gamma^\mu G_\mu^a - \frac{\kappa}{4m_t} \sigma^{\mu\nu} G_{\mu\nu}^a - \frac{i\tilde{\kappa}}{4m_t} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}^a \right] u_t, \quad (2.4)$$

where  $g_s$ ,  $T^a$  and  $m_t$  are the strong coupling constant, SU(3) color matrices and top quark mass, respectively.  $G_{\mu\nu}^a$  means the gluon field strength and  $\sigma^{\mu\nu} \equiv i/2[\gamma^\mu, \gamma^\nu]$ . The  $\kappa$  and  $\tilde{\kappa}$  are the chromomagnetic and chromoelectric coupling, respectively. Though many works have been done so far on the anomalous chromomagnetic and chromoelectric dipole couplings [3, 8, 11, 12, 13], those authors have treated them as real parameters because they have focused only on the effective Lagrangian whose dimension is smaller than or equal to five. However, since in general the anomalous chromomagnetic and chromoelectric dipole moments can be originated from some loop corrections, they can also have imaginary parts. Therefore, we will treat them as complex numbers in our calculations. From Eq. (2.4), we can derive the effective

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<sup>2</sup>At the collider energy which we focus in this work, it is a good approximation to ignore the  $b$  quark mass. Therefore, charged currents for  $b$  quarks become pure  $V - A$ , where  $V$  and  $A$  mean vector and axial currents, respectively, and hence  $b$  quarks are treated as left-handed particles.

<sup>3</sup>For example,  $t(-)$  and  $b(+)$  denote the left-handed top quark and right-handed bottom quark, respectively.

couplings of  $t\bar{t}g$  interaction:

$$\Gamma_{t\bar{t}g} = -ig_s T^a \bar{v}(p_{\bar{t}}, s_{\bar{t}}) \left[ \gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_t} q_\nu (\kappa + i\tilde{\kappa}\gamma^5) \right] u(p_t, s_t), \quad (2.5)$$

where  $p_t(p_{\bar{t}})$ ,  $s_t(s_{\bar{t}})$  and  $q_\nu$  denote four-momenta of the top (anti-top) quark, spin vector of the top (anti-top) quark and incoming gluon momentum, respectively. Though a  $ggt\bar{t}$  four-point interaction is also induced as a result of gauge invariance, we can neglect such an interaction because top quark pair is dominantly produced through quark–anti-quark annihilation processes at the upgraded Tevatron energy. Furthermore, since we assume that there is no new physics except for the interaction related to the top quark, we use the standard form of quark–gluon couplings for ordinary quarks:

$$\Gamma_{q\bar{q}g} = -ig_s T^a \bar{v}(p_b) \gamma^\mu u(p_a), \quad (2.6)$$

where  $p_a(p_b)$  is four-momentum of quarks from the initial proton (anti-proton).

It is remarkable that our approach does not depend on the specific models because we treat the anomalous chromomagnetic and chromoelectric couplings as free parameters. As is well known, in the SM these couplings are estimated to be too small to be detected at the Tevatron. Therefore, if we detect the signal of these couplings, it can be a good evidence of new physics beyond the SM.

### 3. CALCULATION AND DISCUSSION

First, let us focus on the  $CP$ -violating observable  $\Delta\sigma$ , i.e., the numerator of Eq. (2.3) defined by

$$\Delta\sigma \equiv [\sigma(p\bar{p} \rightarrow t(-)\bar{t}(-)X) - \sigma(p\bar{p} \rightarrow t(+)\bar{t}(+)X)], \quad (3.1)$$

where  $\sigma(p\bar{p} \rightarrow \dots)$  denotes the cross section of top pair production for each helicity state at the Tevatron. Furthermore,  $\Delta\sigma$  is given by

$$\begin{aligned} \Delta\sigma = & \sum_q \int_0^1 dx_a \int_0^1 dx_b \int_{-1}^1 d\cos\hat{\theta} f_{q/p}(x_a) f_{\bar{q}/\bar{p}}(x_b) \\ & \times \Theta(x_a x_b s - 4m_t^2) \frac{d\Delta\hat{\sigma}}{d\cos\hat{\theta}} J, \end{aligned} \quad (3.2)$$

where the sum runs over quark flavors;  $q = u, d, c, s, b$ <sup>4</sup>.  $f_{q/p}(x_a)$ ,  $f_{\bar{q}/\bar{p}}(x_b)$  and  $\Theta$  are parton distribution functions and usual step function, respectively. By using Eqs. (2.5-2.6), the subprocess cross section,  $d\Delta\hat{\sigma}/d\cos\hat{\theta}$ , is calculated as

$$\begin{aligned} \frac{d\Delta\hat{\sigma}}{d\cos\hat{\theta}} = & \frac{\pi\alpha_s^2\beta_t}{9\gamma_t m_t \hat{s}^2 \sqrt{\hat{s}}} \left[ \text{Im}\tilde{\kappa} \left\{ (2 - \beta_t^2) \hat{s} \cos^2 \hat{\theta} - 4m_t^2 \right\} \right. \\ & \left. - \text{Im}(\kappa^* \tilde{\kappa}) \hat{s} (1 - 2\cos^2 \hat{\theta}) \right], \end{aligned} \quad (3.3)$$

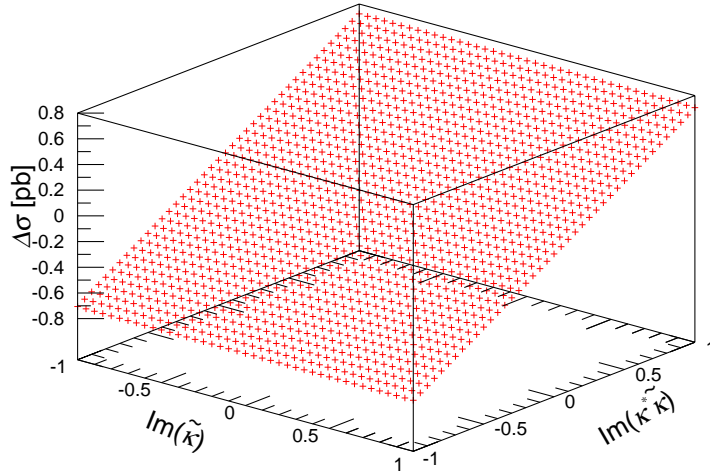
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<sup>4</sup>We assume that top quarks do not exist in the proton as partons.

$\Delta\sigma$		$\text{Im}(\tilde{\kappa})$						
[Pb]		1.0	0.1	0.01	0	-0.01	-0.1	-1.0
$\text{Im}(\kappa^*\tilde{\kappa})$	1.0	$7.06\times 10^{-1}$	$7.13\times 10^{-1}$	$7.14\times 10^{-1}$	$7.14\times 10^{-1}$	$7.14\times 10^{-1}$	$7.15\times 10^{-1}$	$7.23\times 10^{-1}$
	0.5	$3.48\times 10^{-1}$	$3.56\times 10^{-1}$	$3.57\times 10^{-1}$	$3.57\times 10^{-1}$	$3.57\times 10^{-1}$	$3.58\times 10^{-1}$	$3.67\times 10^{-1}$
	0.1	$6.28\times 10^{-2}$	$7.06\times 10^{-2}$	$7.13\times 10^{-2}$	$7.14\times 10^{-2}$	$7.15\times 10^{-2}$	$7.23\times 10^{-2}$	$8.00\times 10^{-2}$
	0.01	$-1.49\times 10^{-3}$	$6.28\times 10^{-3}$	$7.06\times 10^{-3}$	$7.14\times 10^{-3}$	$7.23\times 10^{-3}$	$8.00\times 10^{-3}$	$1.58\times 10^{-2}$
	0	$-8.62\times 10^{-3}$	$-8.62\times 10^{-4}$	$8.62\times 10^{-5}$	0	$8.62\times 10^{-5}$	$8.62\times 10^{-4}$	$8.62\times 10^{-3}$
	-0.01	$-1.58\times 10^{-2}$	$-8.00\times 10^{-3}$	$-7.23\times 10^{-3}$	$-7.14\times 10^{-3}$	$-7.06\times 10^{-3}$	$-6.28\times 10^{-3}$	$-1.49\times 10^{-3}$
	-0.1	$-8.00\times 10^{-2}$	$-7.23\times 10^{-2}$	$-7.15\times 10^{-2}$	$-7.14\times 10^{-2}$	$-7.13\times 10^{-2}$	$-7.06\times 10^{-2}$	$-6.28\times 10^{-2}$
	-0.5	$-3.67\times 10^{-1}$	$-3.58\times 10^{-1}$	$-3.57\times 10^{-1}$	$-3.57\times 10^{-1}$	$-3.57\times 10^{-1}$	$-3.56\times 10^{-1}$	$-3.48\times 10^{-1}$
	-1.0	$-7.23\times 10^{-1}$	$-7.15\times 10^{-1}$	$-7.14\times 10^{-1}$	$-7.14\times 10^{-1}$	$-7.14\times 10^{-1}$	$-7.13\times 10^{-1}$	$-7.06\times 10^{-1}$

**Table 1:** The dependence of the polarization symmetry  $\Delta\sigma$  [pb] on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .

where  $\hat{s} \equiv x_a x_b s$ ,  $\beta_t \equiv \sqrt{1 - 4m_t^2/\hat{s}}$ ,  $\gamma_t \equiv \sqrt{1 - \beta_t^2}$ .  $\hat{\theta}$  denote the emission angle of the top quark in the parton center-of-mass system which is defined in Appendix A. In addition,  $J$  means the Jacobian which transforms the variable  $\hat{t}$  to  $\cos \hat{\theta}$ , given as  $J = |\hat{s}\beta_t/2|$ . Notice that possibilities of the  $CP$ -violation roughly depend only on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  in the  $d\Delta\sigma/d\cos \hat{\theta}$ . Therefore, it is possible to measure  $CP$ -violating observable  $\Delta\sigma$  in some combinations of the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .



**Figure 2:** Surface plots displaying the dependence of  $\Delta\sigma$  on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  at  $\sqrt{s}=2.0$  TeV.

We show the dependence of  $\Delta\sigma$  on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  at the upgraded Tevatron Energy ( $\sqrt{s}=2.0$  TeV) in Fig. 2. In addition, the magnitude of  $\Delta\sigma$  for some combinations of the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  are listed in Table 1. In our numerical calculation, we used the parton distribution functions of CTEQ5M [14] and as input data  $m_t=174.3$  GeV [15]. In Fig. 2, we can easily see that  $\Delta\sigma$  strongly depends on  $\text{Im}(\kappa^*\tilde{\kappa})$  though we cannot individually determine these values. This dependence can be also seen in the same approach to  $CP$ -violation for  $e^+e^- \rightarrow t\bar{t}g$  [16].

Secondly, let us estimate the  $CP$ -violating observable  $\mathcal{A}_{cp}$  which is defined by Eq. (2.3);  $\mathcal{A}_{cp} \equiv \Delta\sigma/\sigma_{\text{total}}$ . Since the total cross section  $\sigma_{\text{total}}$  calculated using (2.4) has many undetermined parameters such as  $|\kappa|^2$ ,  $|\tilde{\kappa}|^2$  and  $\text{Re}(\kappa)$ , we approximated  $\sigma_{\text{total}}$  by  $\sigma_{SM}$  predicted by the SM. This approximation is not unreasonable because the cross section given by the SM is in good agreement with experimental result. In order to observe  $\mathcal{A}_{cp}$  at the 90% Confidence Level (CL),  $\mathcal{A}_{cp}$  must satisfy the condition:

$$|\mathcal{A}_{cp}| \geq \frac{1.64}{\sqrt{N_{\text{events}}}}, \quad (3.4)$$

where  $N_{\text{events}}$  means the total number of events which can be experimentally reconstructed.

At the upgraded Tevatron with center-of-mass energy  $\sqrt{s} = 2.0$  TeV, the decay mode which is most sensitive to the helicities of the produced top and anti-top quark, is the dilepton mode,  $t\bar{t} \rightarrow b\bar{b}W^+(\rightarrow l^+\nu_l)W^-(\rightarrow l^-\bar{\nu}_l)$ , because the energies of leptons produced from  $W$  bosons and  $b$  ( $\bar{b}$ ) are sensitive to the helicities of the produced top and anti-top quark as mentioned in Section 2. The total number of events of the dilepton mode are expected to be detected about 1200 events [9]<sup>5</sup>. Therefore, Eq. (3.4) becomes

$$|\mathcal{A}_{cp}| = \left| \frac{\Delta\sigma}{\sigma_{SM}} \right| \geq \frac{1.64}{\sqrt{N_{\text{events}}}} = \frac{1.64}{\sqrt{1200}}. \quad (3.5)$$

Then, with  $\sigma_{SM} = 7.5\text{pb}$  [9], the  $\Delta\sigma$  should satisfy the relation:

$$|\Delta\sigma| \geq 0.35[\text{pb}]. \quad (3.6)$$

In Table 2, we present the magnitude of  $\mathcal{A}_{cp}$  at 90 % CL for the typical combinations of the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . It is interesting that we can observe the  $\mathcal{A}_{cp}$  at 90 % CL when  $|\text{Im}(\kappa^*\tilde{\kappa})| > 0.5$ , even if  $\text{Im}(\tilde{\kappa})=0$ , as shown in Table 2.

Even if we do not observe signals of  $\mathcal{A}_{cp}$  at the upgraded Tevatron, we can obtain the constraint of the  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .

## 4. SUMMARY AND OUTLOOK

Expecting the observation of  $p\bar{p} \rightarrow t\bar{t}X$  process at the upgraded Tevatron, we estimated the magnitude of  $CP$ -violation effect by using an effective Lagrangian which includes the anomalous chromomagnetic and chromoelectric coupling. Our analysis does not depend on the specific models because we took the anomalous chromomagnetic and chromoelectric coupling as a free parameter. We found that  $CP$ -violation

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<sup>5</sup>In this work, we focus only on Run III (TeV 33) experiment due to the statistical advantage, though there is also Run II experiment.



$\Delta\mathcal{A}_{cp}$ [ $\times 10^{-2}$ ]		$\text{Im}(\tilde{\kappa})$						
		1.0	0.1	0.01	0	-0.01	-0.1	-1.0
$\text{Im}(\kappa^*\tilde{\kappa})$	1.0	9.41	9.51	9.52	9.52	9.52	9.52	9.63
	0.7	6.55	6.65	6.66	6.67	6.67	6.68	6.78
	0.5	$\times$	4.75	4.76	4.76	4.76	4.77	4.88
	0.1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
	0	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
	-0.1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
	-0.5	-4.68	-4.77	-4.76	-4.76	-4.76	-4.75	$\times$
	-0.7	-6.78	-6.68	-6.67	-6.67	-6.66	-6.65	-6.55
	-1.0	-9.63	-9.52	-9.52	-9.52	-9.52	-9.51	-9.41

**Table 2:** The expected magnitude of the  $\mathcal{A}_{cp}$  at the Upgraded Tevatron. A symbol “ $\times$ ” means that the magnitude of the  $\mathcal{A}_{cp}$  are smaller than that at 90% CL as is defined by Eq.(3.4).

depends only on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . Especially, dependence of  $\text{Im}(\kappa^*\tilde{\kappa})$  is stronger than that of  $\text{Im}(\tilde{\kappa})$  in this process. Furthermore, we pointed out that  $CP$ -violating observables,  $\mathcal{A}_{cp}$ , can be measured at 90 % CL in some combinations of the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .

Though the same analysis can be applied for the Large Hadron collider (LHC) at CERN which will start in 2005, the behavior of the  $CP$ -violation may be different from this analysis because in this case the dominant process of top quark pair production is two-gluon fusion which was neglected in this work.

Since this analysis was done without specific models, the model could not be specified from this analysis, even if the  $CP$ -violation is measured in this process. However, such measurements could give the constraints on parameters in any models.

Finally, we did not analyze decays of top quarks in process, which needs further investigation. The analysis of  $CP$ -violation at the LHC is also important and will be done further in future work.

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## A. THE SPIN VECTORS AND FOUR-MOMENTA IN PARTON CENTER-OF-MASS SYSTEM

The definition of the spin vectors and four-momenta in the parton center-of-mass system is given in this appendix.

We define the spin four-vectors,  $s_t, s_{\bar{t}}$ , in the parton center-of-mass system (CMS) in terms of a spin angle  $\xi$  as it is illustrated in Ref [17].

$$s_t = \frac{1}{\gamma_t}(\beta_t \cos \xi; \gamma_t \sin \xi, 0, -\cos \xi) \quad (\text{A.1})$$

$$s_{\bar{t}} = \frac{1}{\gamma_t}(\beta_t \cos \xi; \gamma_t \sin \xi, 0, \cos \xi) \quad (\text{A.2})$$

with  $\beta_t \equiv \sqrt{1 - 4m_t^2/\hat{s}}$ ,  $\gamma_t \equiv \sqrt{1 - \beta_t^2}$ .

Here, we set as  $\xi=\pi$  because we calculated in the helicity basis.

In the parton CMS with the z-axis chosen to be along the top quark direction of motion, the four-momenta read as follows:

$$\begin{aligned} p_t &= \frac{\sqrt{\hat{s}}}{2}(1; 0, 0, \beta_t) \\ p_{\bar{t}} &= \frac{\sqrt{\hat{s}}}{2}(1; 0, 0, -\beta_t) \\ p_a &= \frac{\sqrt{\hat{s}}}{2}(1; \sin \hat{\theta}, 0, \cos \hat{\theta}) \\ p_b &= \frac{\sqrt{\hat{s}}}{2}(1; -\sin \hat{\theta}, 0, -\cos \hat{\theta}), \end{aligned} \quad (\text{A.3})$$

where  $\hat{\theta}$  denotes the emission angle of the top quark.

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